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MOTION OF A SPHERE IN A FLUID DUE TO GRAVITY

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The motion of a body in a fluid is one of the most important problems in hydrodynamics. Since a sphere is the simplest body of high symmetry, many effects and phenomena occurring during its motion take a clear form. The literature devoted to the motion of a free sphere or flow past a fixed sphere is quite voluminous. A review of classical works (beginning with Newton's experiments) can be found in [1–5].

The problem discussed involves great difficulties for theoretical analysis. In an ideal fluid model d'Alembert's paradox exists. This paradox can be eliminated by taking fluid viscosity into account. But even in this case, analytical solutions can be obtained only for very small Reynolds numbers. The most fundamental solution was found by Stokes [6], and further progress is very difficult [2, 4]. The main impediment to theoretical analysis is that, under certain conditions, the motion of a free sphere or flow past a fixed sphere become unstable even against infinitesimal perturbations, and a number of fundamental postulates that are used in theoretical analysis become inapplicable. The principle of flow reversal and an *a priori* assumption on the type of flow symmetry are among such postulates.

The first experiments described in detail in [1, 2] showed the great difference between the motion of a free sphere and flow past a fixed sphere at fairly great Reynolds numbers. Particularly, it has been found that a freely submerging sphere can deviate from a straight trajectory, and its drag coefficient can exceed considerably that of a fixed sphere. In [7], using the impressive phenomenon of rectilinear floating of air bubbles in water, attention is drawn to the physical inconsistency of a priori assumptions of the symmetry of solutions. In recent years, the question of symmetry and symmetry-breaking bifurcations has been actively studied [8].

The results of the experiments of [9] are concerned with one of the most fundamental postulates according to which two dynamic systems behave in a similar manner if a few of their first integrals of motion are identical, and the possible differences between the other integrals are not significant.

Experiments with a sphere have produced many other unexpected results. First, the crisis of the drag at Reynolds numbers of the order of $2 \cdot 10^5$, which is explained from a physical point of view in [10, 11], deserves mention. The results of experiments in a vibrating fluid [12], in which motion against gravity was observed, are very impressive, as are those of experiments of Taylor [13] with a sphere in a rotating fluid. Excellent photographs of flow in a wake behind a fixed sphere were obtained by Taneda and Gudkov [14–16]. The experiments of Mowbray and Rarity [17] made a significant contribution to the study of the effect of density stratification.

In [18] and in other papers of the same authors, attention is focused on the effects associated with molecular diffusion and a wake behind a fixed sphere is used as an example. A sphere is often used in studies of sediment transport, high-speed entry of bodies into water, and other important applied problems.

In recent years numerical experiments have made a certain contribution to the solution of the complicated problem of motion of a sphere in a fluid. As an example, we mention the works of Kim et al. [5, 10], which are of interest, because they are devoted to the central question, namely, to the analysis of the stability of motion. At present an analysis has been performed only for a wake behind a fixed sphere. The fundamental difference of the problem of the stability of motion of a free sphere is that this problem is a

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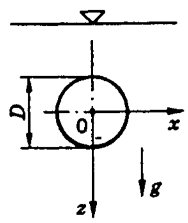

Fig. 1

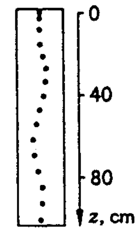

Fig. 2

two-parameter one. In addition, in this case there is a relationship between displacements of the sphere and the forces acting on it, and the energy redistribution between the six degrees of freedom is also possible.

This work is intended to attract attention to the interesting problem of the stability of immersion of a free sphere by gravity and to supplement the available experimental data [1, 2, 19–22] which are still limited. Similar effects are observed during the floating of gas bubbles in a fluid [23] and in motion of drops of one fluid in another [24].

A schematic diagram of the experiments is shown in Fig. 1. An ebonite sphere of diameter D was suspended by a thin thread in a transparent vessel with a 45×45 cm square cross-section and with a height of 1.05 m. The vessel was filled with distilled water so that the sphere was completely immersed in water. The sphere density ρ_1 was greater than the water density ρ by a factor of 1.2. The acceleration of gravity g and the kinematic viscosity coefficient of water ν are two dimensional parameters that also play an important role.

At $t = 0$, when both the liquid and the sphere were at rest, the thread was carefully released, and free immersion of the sphere began. The projection of its trajectory onto the plane (x, z) which is parallel to one of the side walls of the vessel was recorded at a speed of 24 frames per second. The z axis was directed downward and the x axis was directed to the right of the observer. The reference point coincided with the center of the sphere at rest. The spatial trajectory of the sphere was also traced visually. To study water flow, we added aluminum powder particles to water, which were so small that they remain suspended in water for a few days.

At rest the gravitational force and the Archimedes force go through the same point which is the center of the sphere, so that the metacentric height is equal to zero. From the point of view of hydrostatics such a state is contiguous to the instability region. During motion of the sphere, the pressure distribution over the sphere surface changes, and inertial and friction forces appear. The inertial force stabilizes the motion, while the pressure and friction may either stabilize or destabilize it. The moving sphere has six degrees of freedom, but the external gravitational force acts along only one of the directions. In such a situation, the effect of small perturbations is difficult to predict.

Only the parameter D was varied in the main experiments. It took values of 2, 3, and 4 cm, which were much smaller than the cross-sectional dimensions of the vessel. Auxiliary experiments were also performed to study the influence of the free surface, strong variations of density ρ_1, significant changes in the shape and the center of gravity of the body, and also the influence of the considerable displacement of the initial position of the sphere toward one of the vessel walls. As a result, the conditions in the main experiment were chosen so that the parameters D, ρ_1, ρ, g, and ν were decisive and the remaining parameters were attributable to small perturbations, which are inevitably the case in practice. Note only that we did not observe deviations from the straight trajectory in auxiliary experiments with a steel sphere 3 cm in diameter and with a plastic sphere whose density did not differ significantly from ρ, i.e., the region of stability of the motion obviously exists. But the main experiments were intended to show that strong deflections of the straight trajectory are also possible.

The results of experiments for $D = 2$, 3, and 4 cm were qualitatively similar. The spheres moved rectilinearly with acceleration along a certain initial section of length l_0. The flows past the spheres were

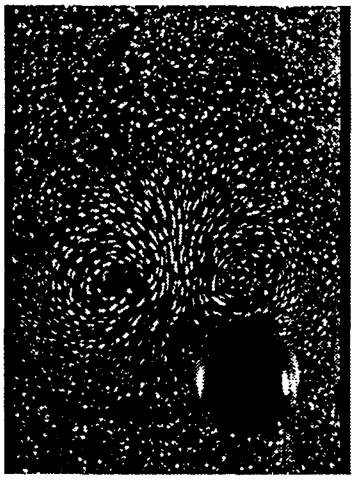

Fig. 3

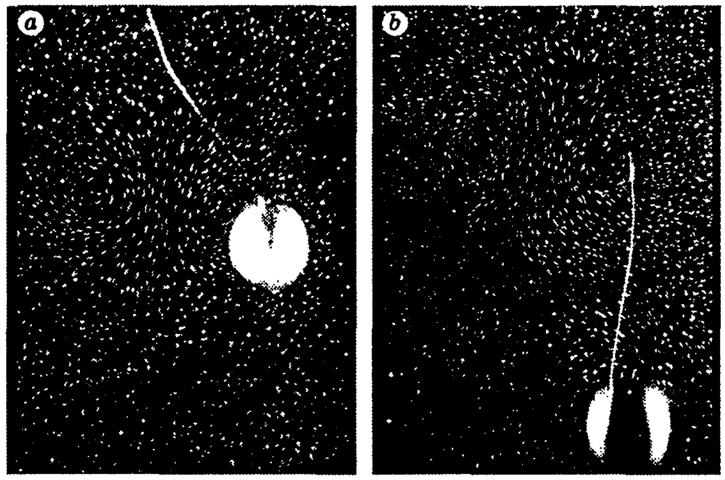

Fig. 4

practically without separation. At the end of the initial section, the sphere velocity decreased noticeably, a vortex separated from the sphere, and a sudden deflection of the straight trajectory occurred. No predominant direction of the deflection was observed. The obtained information is not sufficient to draw well-founded conclusions on the trajectory shape after the loss of stability. The trajectory looked random.

The separated vortex quickly lagged behind the sphere, and the hydrodynamic wake again narrowed. After a time, however, another vortex separated, and another change in the direction of motion of the sphere was observed. In the experiment the spheres were well away from the vessel walls, so that the result obtained is very likely true for an unbounded fluid. The vertical dimension of the vessel enabled not more than three separated vortices to be observed.

As with a fixed sphere [16], acceleration at the initial section results in delayed flow separation. But the quantitative measure of this effect was unexpectedly great. For a sphere with a diameter of 4 cm, separation of the first vortex occurred at a Reynolds number of about 14,000. At this Reynolds number the flow in the wake behind the fixed sphere becomes turbulent. At the time of separation of the first vortex, the immersion velocity of the free sphere practically attains a limit.

All the aforesaid is illustrated by photographs and graphs. The projection of the trajectory of the sphere with a diameter of 2 cm onto the plane (x, z) was filmed and processed by a computer, and the result is shown in Fig. 2. The main idea of the processing consisted in finding the function $x(z)$, which is shown in Fig. 2, while the film contained data on the trajectory in the form of the parametric functions $x(t)$ and $z(t)$. The time interval between the neighboring sphere images was 1/6 sec. The upper boundary of the frame in Fig. 2 coincides with the free water surface. The uppermost image of the sphere was taken for $z = 0$, and the lowermost image for $z = 90$ cm. The distance between the side boundaries of the frame in Fig. 2 equals 18.6 cm and is considerably smaller than the cross-sectional dimensions of the vessel. The strong retardation of the sphere before deviation from the vertical trajectory results in a noticeable change in the interval Δz between the neighboring images in Fig. 2.

The flow pattern for deviation of a 4-cm diameter sphere from the straight trajectory is shown in Fig. 3. The exposure was 1/30 sec. The light dots and lines are images of the particles marks of aluminum powder. In the fluid at rest the particles produce images in the form of dots, but in the moving fluid they form tracks in a photograph. Before deviation the sphere moved in the middle between the side boundaries of the frame. At the record time, the sphere velocity was about 35 cm/sec.

The photograph primarily confirms that there was practically no wake behind the sphere before vortex separation, because no tracks are observed in this region of the image. The separated vortex remained on the

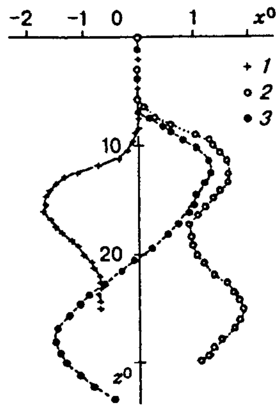

Fig. 5

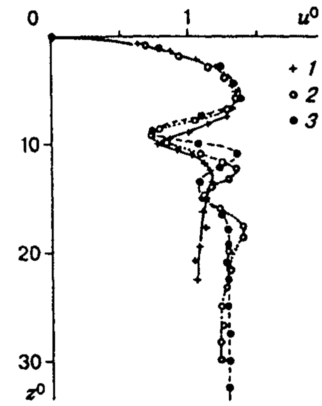

Fig. 6

straight motion path, and the sphere deviated to the right practically in the observation plane. The latter follows from the illumination pattern of the sphere by the "light knife" employed in the method of visualization by aluminum powder. The "knife" thickness was about 1 cm.

Figure 4 gives two successive photographs of experiments with a 3-cm diameter sphere. The time interval between these two frames is not known exactly. It can be evaluated using information given in the graphs below. The sphere initially deviated to the right moving away from the observer (Fig. 4a), and then it smoothly turned to the left approaching the observer (Fig. 4b). The second vortex had not yet formed, and the first vortex lagged behind the sphere.

The length of thread used to suspend the sphere is also seen in Fig. 4. This supplements the information on the motion of the sphere. At the same time there are strong grounds to believe that the thread did not have significant influence on the loss of stability. In particular, we performed experiments in which a large enough steel bolt was bonded to the sphere. This shifted considerably the location of the center of gravity and changed the conditions of flow past the sphere. Nevertheless, the process of the loss of stability did not change qualitatively.

Among the five dimensional parameters mentioned above, only two independent dimensionless complexes can be formed, although there is a great number of complexes which transform into each other uniquely. One of the dimensionless parameters is chosen in the form $\varepsilon = (\rho_1 - \rho)/\rho$. The set of variants of the second complex can be written as

$$A = (gD^3/\nu^2)^n f, \tag{1}$$

where f is an arbitrary dimensionless function, and n is an arbitrary constant.

Three special cases of n are of interest: $n_1 = 1/3$, $n_2 = 1/2$, and $n_3 = 1$. For $n = n_1$, the dimensionless Navier–Stokes equations do not contain any parameters; each of their coefficient is equal to 1 or 1/2. An advantage of this choice is that the part of the operator that is common to many systems with a fluid becomes universal. Dimensionless parameters appear only in the initial and boundary conditions and in the characteristics of introduced perturbations. With such a choice of n the characteristic length and time scales have the form

$$L_1 = (\nu^2/g)^{1/3}, \quad T_1 = (\nu/g^2)^{1/3}.$$

The variant $n = n_2$ is obtained if one assumes that sooner or later the sphere enters a stationary uniform motion regime, as in the solution by Stokes. In this case, the single parameter $\mathrm{Re} = (gD^3/\nu^2)^{1/2}$ remains in the Navier–Stokes equations, and the characteristic length and time scales are as follows: $L_2 = D$ and $T_2 = (D/g)^{1/2}$. The variant $n = n_3$ is obtained for to $L_3 = D$ and $T_3 = D^2/\nu$. Under these conditions, only the parameter $\mathrm{Ar} = gD^3/\nu^2$ remains in the Navier–Stokes equations.

The available information does not enable one to know which of the variants is preferred, and, below,

TABLE 1

D, cm	u_∞, cm/sec	$u_\infty D/\nu$	c_z
2	26	5200	0.77
3	30	9,000	0.87
4	30	12,000	1.16

as the characteristic length and time scales we use $L = D$ and $T = (D/\varepsilon g)^{1/2}$. Here $\varepsilon^{1/2}$ plays the role of the function f.

Examples of the functions $x^0(z^0)$ and $u^0(z^0)$ are shown in Figs. 5 and 6, where u is the vertical component of the sphere velocity. The superscript 0 denotes conversion to dimensionless values using L and T, and points 1–3 correspond to the results of experiments for $D = 4$, 3, and 2 cm, respectively. The graphs show that, after the loss of stability, the vertical velocity of the sphere tends to a constant value after a few decaying oscillations. For the function $x^0(z^0)$, the data obtained are not sufficient to draw any well-founded conclusions.

It is useful to consider the role of the symmetry of the sphere and of the fluid viscosity. The inertia of the sphere is essential for the translational degrees of freedom. At the same time, the inertia of the sphere does not influence the rotational degrees of freedom because of symmetry. Fluid viscosity can prevent rotation. But the stabilizing effect of viscosity manifests itself slowly. Moreover, viscosity can also disturb flow stability. The destabilizing role of viscosity was first substantiated theoretically in the problem of stability of plane-parallel shear flows. Allowance for fluid viscosity in the corresponding mathematical model has led to the discovery of Tollmien–Schlichting waves, which play a decisive role in laminar-turbulent transition. Benjamin [25] and Stepanyants and Fabrikant [26] have shown that this example is not unique.

In the presence of instability the problem of a universal description of the system's response to the introduced perturbation cannot be easily solved. Nevertheless, for the vertical velocity component, certain universality is observed even with the relatively simple method of normalization adopted here: the three curves in Fig. 6 coincide well enough in the region of rectilinear motion up to the location and value of their first maximum.

To extend the region of universality, one should use a more complex function f in (1). This situation is typical for all problems of hydrodynamics. For example, in the stationary problem of flow past a fixed body the drag and lift coefficients are used which can be treated as f. In the nonstationary case, they are supplemented by the added mass coefficients. The coefficients of loss in head play a similar role in hydraulics. These coefficients can be calculated only for the laminar flow regime. They are determined experimentally over a wider range of parameters.

In the problem discussed, the drag coefficient c_z can be determined using the asymptotic value of the longitudinal velocity component u_∞ at large t. In this case, from the balance of forces in the projection onto the z axis we obtain

$$c_z = 4\varepsilon g D/(3u_\infty^2). \tag{2}$$

This algorithm was used by Schiller [2], who obtained a considerable excess over the drag coefficient of a fixed sphere. The results of processing of the data in Fig. 6 using algorithm (2) are given in Table 1. These results are consistent with the conclusions of [2] that c_z depends strongly on the Reynolds number and exceeds considerably the corresponding value for a fixed sphere. When D and u_∞ are used to determine the Reynolds number, the value of c_z for a fixed sphere is almost constant and is 0.42 in the range discussed.

A drawback of the algorithm (2) is that it takes only one velocity component into account. The question of whether the drag coefficients of fixed and free spheres agree if the latter is determined from the velocity magnitude still remain to be solved.

The extensive literature on hydrodynamical stability deals mainly with problems with one parameter (such as the Reynolds number). The free motion of a sphere is of interest, because it is a problem of practical importance with two parameters: ε and A. Another two-parameter flow was studied in [27]. The problem of location of the boundary of the instability region in the plane of two parameters is important. In [27] this boundary was very simple and was found experimentally. The situation with a moving sphere is more complicated. Particularly, the question of whether the processes of immersion and flotation of spheres which differ only in the sign of ε are identical is not a trivial question. In this case, it is tempting to postulate the existence of a corresponding symmetry. However, one might expect that this postulate would be inapplicable under certain conditions.

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